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1998 J. Phys. A: Math. Gen. 31 L233

(http://iopscience.iop.org/0305-4470/31/12/001)

J. Phys. A: Math. Gen. **31** (1998) L233–L239. Printed in the UK PII: S0305-4470(98)87550-1

LETTER TO THE EDITOR

New integrable generalization of the one-dimensional *t–J* model

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Received 12 September 1997, in final form 18 December 1997

Abstract. A new generalization of the *t–J* model with a nearest-neighbour hopping is formulated and solved exactly by the Bethe ansatz method in the thermodynamic limit. The model describes the dynamics of fermions with different spins and with isotropic and anisotropic interactions.

Recently there has been considerable interest in studying low-dimensional electronic models of strong correlation due to the possibility that the normal state of the two-dimensional novel superconductivity may share some interesting features of a one dimensional interacting electron system [1]. In one-dimension, the Bethe ansatz technique can allow one to exactly solve Hamiltonians in special cases, such as the Hubbard model [2] and the ordinary *t–J* model at its supersymmetric point [3, 4]. For example, it is possible to obtain the low-energy gapless excitation spectrum around the ground state by the finite-size scaling method [5, 6] and calculate the critical exponents of the correlation functions [7–9].

The *t–J* model is a lattice model on the restricted electronic Hilbert space, where the occurrence of two electrons on the same lattice site is forbidden. This restriction corresponds to an implicitly infinite on-site Coulomb repulsion. Two types of interactions between electrons on nearest-neighbour sites are considered: a change interaction of strength V and a spin-exchange interaction J. The Hamiltonian of the extended version of the *t–J* model with spin $S = (N-1)/2$ has the form [3, 4, 10]

$$H = -\sum_{j=1}^{L}\sum_{\alpha=1}^{N}\mathcal{P}(c_{j,\alpha}^{+}c_{j+1,\alpha} + c_{j+1,\alpha}^{+}c_{j,\alpha})\mathcal{P} - \sum_{j=1}^{L}\left[J\sum_{\alpha\neq\beta}^{N} c_{j,\alpha}^{+}c_{j,\beta}c_{j+1,\beta}^{+}c_{j+1,\alpha} + \sum_{\alpha,\beta=1}^{N} V_{\alpha\beta}c_{j,\alpha}^{+}c_{j,\alpha}c_{j+1,\beta}^{+}c_{j+1,\beta}\right] \tag{1}$$

where $c_{j,\alpha}$ annihilates an electron with a spin component α, $\mathcal{P}$ is the projector on the subspace of non-doubly occupied states, and L is the lattice size. The anisotropy in the charge interactions is introduced through a matrix $V_{\alpha\beta}$.

In the isotropic case $V_{\alpha\beta} = V$ the Hamiltonian (1) corresponds to the traditional *t–J* model which was exactly solved by the Bethe ansatz method at the supersymmetric point ($V = -J = 1$) for the case $S = \frac{1}{2}$ [3, 4, 9–11]. The generalization of this result for the

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0305-4470/98/120233+07$19.50

arbitrary spin S was carried out in [12–14]. Other generalizations of the t–J model were studied in [15–18]. In particular, in [18] it was shown that the model (1) is solvable for arbitrary spin and special values of the coupling J and $V_{\alpha\beta}$:

$$\begin{aligned} &J = \varepsilon_0 \\ &V_{\alpha\beta} = -\varepsilon_0\{(1+\varepsilon_\alpha)\cosh\gamma\cdot\delta_{\alpha\beta} + \exp[\mathrm{sign}(\alpha-\beta)\gamma](1-\delta_{\alpha\beta})\} \end{aligned} \tag{2}$$

where $\gamma > 0$ is a measure of the anisotropy and $\epsilon_0, \epsilon_\alpha = \pm 1 (\alpha = 1, \ldots, N)$. It was shown more exactly in [18] that the Hamiltonian (1), (2) is the quantum counterpart of the so-called Perk–Schultz model [19] which was diagonalized by Schultz in the most general form [20] (see also [21, 22]). This model is also related to the anisotropic version of the Sutherland model [11]. In fact, in [19] Perk and Schultz considered two models, the Hamiltonian of the first one, in terms of fermionic creation operators, is given by (1), (2) and the Hamiltonian of the second one is given by [19, 22]

$$H = -\sum_{j=1}^{L}\sum_{\alpha,\beta=1}^{N} c_{j\alpha}^{+} c_{j\beta} c_{j+1,\alpha}^{+} c_{j+1,\beta}. \tag{3}$$

The systems which are described by (3) and their different modifications were studied in their spin formulation in [23–25] (see also references therein). In this letter we present a new set of models of strongly correlated particles which are exactly solvable. In these models we have different classes of particles. The interactions among these particles are of type (1) or (3) depending on whether the particles belong to the same class.

First let us formulate the problem and write down the Hamiltonian. Consider a periodic chain with L sites and n particles. There are N possible types (or species) of particles labelled by $\alpha = 1, 2, \ldots, N$. These distinct types of particles are split in P classes, $Q_j (j = 1, 2, \ldots, P)$, each class having $N_{Q_j} (j = 1, 2, \ldots, P)$ types of particles. This implies

$$\sum_{i=1}^{P} N_{Q_i} = N$$

and if, in a given configuration, n_{Q_j} is the total number of particles in classes Q_j we also have

$$\sum_{i=1}^{P} n_{Q_j} = n.$$

We order the labels that specify the type of particle sequentially such that the first N_{Q_1} of them are the types in class Q_1, the next N_{Q_2} in class Q_2 and so on. This allows us to use the simplified notation $\sum_{\alpha\epsilon Q_i}$ instead of

$$\sum_{\alpha=N_{Q_1}+\cdots+N_{Q_{i-1}}+1}^{N_{Q_1}+\cdots+N_{Q_i}}.$$

In each class where $N_{Q_i} > 1$ we choose an arbitrary one-to-one correspondence between the particle types $\alpha \Leftrightarrow \bar{\alpha}(\alpha, \bar{\alpha} \in Q_i), (i = 1, 2, \ldots, P)$. So in each class there are q_{Q_j} conjugate species, where

$$[(N_{Q_i}+1)/2] \leqslant q_{Q_i} \leqslant N_{Q_i}$$

and $[N/2]$ means the integer part of the number $N/2$. In the case where $N_{Q_i} = 1$, since we have only one type of particles there are no conjugate species of particles. The dynamics

of the above-defined particles is described by the Hamiltonian

$$\begin{aligned} H = &-\sum_{j=1}^{L}\sum_{\alpha=1}^{N}\mathcal{P}(c^{+}_{j,\alpha}c_{j+1,\alpha}+c^{+}_{j+1,\alpha}c_{j,\alpha})\mathcal{P} \\ &-\sum_{j=1}^{L}\sum_{i=1}^{P}\sum_{\alpha,\beta\in Q_i}\varepsilon_0\varepsilon_i[U^{(i)}_{\alpha}U^{(i)}_{\beta}c^{+}_{j\alpha}c_{j\beta}c^{+}_{j+1,\bar{\alpha}}c_{j+1,\bar{\beta}}-(1+\varepsilon_i)\cosh\gamma n_{j\alpha}n_{j+1,\beta}] \\ &-\sum_{j=1}^{N}\sum_{i\neq k=1}^{P}\sum_{\alpha\in Q_i,\beta\in Q_k}\varepsilon_0\{g_{ik}c^{+}_{j\alpha}c_{j\beta}c^{+}_{j+1,\beta}c_{j+1,\alpha}-\exp[\mathrm{sign}(\alpha-\beta)\gamma]n_{j\alpha}n_{j+1,\beta}\} \end{aligned} \tag{4}$$

where $\varepsilon_i, \varepsilon_0 = \pm 1$, $g_{ij} = g_{ji}^{-1}$ and the parameters $U^{(i)}_{\alpha}$ play the role of anisotropies inside of each set Q_i and satisfy

$$U^{(i)}_{\alpha} = 1/U^{(i)}_{\bar{\alpha}} \qquad \sum_{\alpha\in Q_i}(U^{(i)}_{\alpha})^2 = 2\cosh\gamma. \tag{5}$$

The ordinary t–J model given in (1), (2) is obtained by choosing $P = N$ and $g_{ik} = 1$. In this case we have only one particle in each class and consequently we have no conjugate species and constraint (5) can be ignored. The anisotropic version of the Perk–Schultz model of the second type (3) can be obtained by choosing $P = 1$ so that all particles belong to the same class. The Hamiltonian (3) is the isotropic version of the model [23, 24] in the sector where we have no holes ($N = L$), and the conjugate species are ($\alpha, \bar{\alpha} = N-\alpha+1$). The case in which we also have holes ($N \leqslant L$) was introduced and studied in [25, 26]. In the general case the Hamiltonian (4) describes the dynamics of fermions with different spins. For each class we have one kind of fermions, which have spin interactions among its components inside of a given class ('internal') and between distinct classes ('external'). For example, the case $N_{Q_i} = 2$ corresponds to spin $S_i = \frac{1}{2}$ and the 'internal' interactions are quadratic while in the case $N_{Q_i} = 3$ we have a spin $S_i = 1$ with biquadratic 'internal' interactions [25]. For the general case $S_i = (N_{Q_i}-1)/2$ the 'internal' magnetic interactions inside of each set can be written as a polynomial of degree $2S_i$ in the spin operator.

The exact solution for the eigenstates and eigenvalues of the Hamiltonian (4) can be obtained within the framework of the Bethe ansatz method [27, 28]. The central object of this method is the two-particle scattering matrix S which is calculated from the single- and two-particle processes described by the Hamiltonian (4). In this model a direct calculation of the scattering processes involving three or more particles shows that these processes can be expressed as the product of two-particles scattering ones. This is a direct consequence of the constraint that we have at most one particle per site, like in the ordinary t–J model [3]. The nonvanishing elements of the S-matrix are given by

$$\begin{aligned} &S^{\alpha\beta}_{\alpha'\beta'}(k_1,k_2) = [\sin(\mathrm{i}\gamma-\lambda_1+\lambda_2)]^{-1}\hat{S}^{\alpha\beta}_{\alpha'\beta'}(\lambda_1-\lambda_2) \\ &\hat{S}^{\alpha\beta}_{\alpha'\beta'}(\lambda) = \delta_{\alpha\beta'}\delta_{\beta,\alpha'}\sin(\mathrm{i}\gamma+\varepsilon_i\lambda)-\varepsilon_i\delta_{\alpha\bar{\beta}}\delta_{\beta'\bar{\alpha}'}U^{(i)}_{\alpha}U^{(i)}_{\beta'}\sin\lambda \qquad \text{for } \alpha,\beta,\alpha',\beta'\in Q_i \\ &\hat{S}^{\alpha\beta}_{\alpha'\beta'}(\lambda) = \mathrm{i}\delta_{\alpha\beta'}\delta_{\beta,\alpha'}\sinh\gamma\exp[\mathrm{i}\,\mathrm{sign}(\beta-\alpha)\lambda]-g_{ik}\delta_{\alpha\alpha'}\delta_{\beta\beta'}\sin\lambda \\ &\qquad\qquad \text{for } \alpha\in Q_i;\ \beta\in Q_k,\ i\neq k \end{aligned} \tag{6}$$

where $\lambda_j (j = 1, 2, \ldots, n)$ are suitable particle rapidities related to the momenta $\{k_j\}$ of the electrons by

$$k_j = \begin{cases} \pi-\Theta(\lambda_j;\frac{1}{2}\gamma) & \varepsilon_0 = -1 \\ -\Theta(\lambda_j;\frac{1}{2}\gamma) & \varepsilon_0 = +1 \end{cases} \tag{7}$$

with the function Θ defined by

$$\Theta(\lambda;\gamma) = 2\arctan(\cot\gamma\cdot\tan\lambda) \qquad -\pi < \Theta(\lambda,\gamma) \leqslant \pi. \tag{8}$$

The S-matrix for the given model has to satisfy the Yang–Baxter equations [27, 29] in order to ensure exact integrability through the Bethe ansatz method. As far as we know, the form of the S-matrix (6) is a new one, therefore it is necessary to check the Yang–Baxter equations. We have checked these equations numerically for the different choices of parameters $U_\alpha^{(i)}$ and the sets Q_i.

A different way to check the Yang–Baxter equations comes from the fact that the S-matrix is related with the quantum chain

$$H = \sum_{j=1}^{N-1} e_j \tag{9}$$

where

$$\begin{aligned} e_j = \cosh\gamma + \sum_{i\neq k}\sum_{\alpha\in Q_i,\beta\in Q_k} [g_{ik}E_j^{\alpha\beta}E_{j+1}^{\beta\alpha} - \sinh\gamma\,\mathrm{sign}(\alpha-\beta)E_j^{\alpha\alpha}E_{j+1}^{\beta\beta}] \\ + \sum_i \sum_{\alpha,\beta\in Q_i} \varepsilon_i [U_\alpha^{(i)}U_\beta^{(i)}E_j^{\alpha\beta}E_{j+1}^{\bar\alpha\bar\beta} - \cosh\gamma\,E_j^{\alpha\alpha}E_{j+1}^{\beta\beta}] \end{aligned} \tag{10}$$

and the $N\times N$ matrices $E^{\alpha\beta}$ have elements $(E^{\alpha\beta})_{\gamma\delta} = \delta_{\alpha\gamma}\delta_{\beta\delta}$. More precisely the matrix $\hat S(U)$ is expressed in terms of the two-body $N^2\times N^2$ matrix e_j by the relation

$$\begin{aligned} \hat S_j(U) &= \sum_{\alpha,\beta\gamma,\delta} \hat S_{\gamma,\delta}^{\alpha,\beta} E_j^{\alpha,\delta}E_{j+1}^{\beta,\gamma} \\ &= \sin(\lambda+\mathrm{i}\gamma) - \sin(\lambda)e_j. \end{aligned} \tag{11}$$

The $\hat S(U)$-matrix satisfies the Yang–Baxter equation if the matrices e_j are generators of the Hecke algebra [30, 31]

$$\begin{aligned} &e_je_{j\pm1}e_j - e_j = e_{j\pm1}e_je_{j\pm1} - e_{j\pm1} \\ &[e_j,e_k] = 0 \qquad \text{for } |j-k|\geqslant 2 \\ &e_j^2 = 2\cosh\gamma\, e_j \end{aligned} \tag{12}$$

since in this case relation (11) is the standard 'Baxterization' of the Hecke generators [31]. We have checked that e_j satisfies equations (12) and thus we proved the integrability of the model (4) [33].

Moreover, the quantum chain (9), (10) is exactly integrable and contains as a particular case the Perk–Schultz and Sutherland chains [19, 11]. It is interesting to observe that the Hamiltonian (9) with $N+1$ types of particles split in $P+1$ groups $Q_j (j = 1, 2, \ldots, P+1)$, in which the first group has only one type of particles ($N_{Q_1} = 1$) and $g_{1k} = g_{k1} = -1 (k = 2, \ldots, N+1)$ reproduces, apart from the boundary term, the Hamiltonian (4) with holes and N types of particles.

The Hamiltonian (4) is diagonalized by a standard procedure by imposing periodic boundary conditions on the Bethe function. These boundary conditions can be expressed in terms of the transfer matrix of the non-uniform models related to (6) by using the quantum method of the inverse problem [32, 33]. The rapidities λ_j that define a n-particle wavefunction are obtained by solving the equations

$$\left[\frac{\sinh(\lambda_j - \mathrm{i}\gamma/2)}{\sinh(\lambda_j+\mathrm{i}\gamma/2)}\right]^L = (-1)^{n-1}\Lambda(\lambda_j) \tag{13}$$

where $\Lambda(\lambda)$ is the eigenvalue of the transfer matrix

$$T^{\{\alpha_l\}}_{\{\alpha'_l\}}(\lambda) = \sum_{\{\beta_l\}} \prod_{l=1}^{n} S^{\alpha_l \beta_{l+1}}_{\alpha'_l \beta_l}(\lambda_l - \lambda) \qquad (\beta_{n+1} = \beta_1). \tag{14}$$

It is simple to verify that besides the number of particles in each class of particles n_{Q_i} the number of conjugate pairs in each class, which we denote by n'_i is also a conserved quantity in the Hamiltonian (4). Here we call two conjugate particles of the same class paired if they are consecutive particles and have no unpaired particles of this class between them. We also denote by $n_i (n_i = n_{Q_i} - 2n'_i)$ the number of unpaired particles in class Q_i. In the general case the complete diagonalization of the transfer matrix (14) is not a simple problem. For simplicity we restrict ourselves to some particular cases. As a starting point let us consider model (4) in the sector where we have no pairs of particles. If we have no pairs of a set Q_i then the first interaction term in the Hamiltonian (4) does not work and all particles of this set are identical and can be considered as one component of the model. In this way the general model (4) is reduced to the anisotropic t–J model with P components [18, 20] and the diagonalization of the transfer matrix of the inhomogenous model (14) gives the following Bethe ansatz equations

$$\prod_{j'=1}^{m_{\sigma-1}} \frac{\sin\left(\lambda_j^{(\sigma)} - \lambda_{j'}^{(\sigma-1)} + \frac{\mathrm{i}}{2}\epsilon_\sigma \gamma\right)}{\sin\left(\lambda_j^{(\sigma)} - \lambda_{j'}^{(\sigma-1)} - \frac{\mathrm{i}}{2}\epsilon_\sigma \gamma\right)} = -\epsilon_\sigma^{n_\sigma} \epsilon_{\sigma+1}^{n_{\sigma+1}} \prod_{\rho=0}^{q} (G_{\sigma\rho} G_{\rho\sigma+1})^{n_\rho}$$
$$\times \prod_{j'=1}^{m_\sigma} \frac{\sin(\lambda_j^{(\sigma)} - \lambda_{j'}^{(\sigma)} + \mathrm{i}\epsilon_{\sigma+1}\gamma)}{\sin(\lambda_j^{(\sigma)} - \lambda_{j'}^{(\sigma)} - \mathrm{i}\epsilon_\sigma \gamma)} \prod_{j'=1}^{m_{\sigma+1}} \frac{\sin\left(\lambda_j^{(\sigma)} - \lambda_{j'}^{(\sigma+1)} - \frac{\mathrm{i}}{2}\epsilon_{\sigma+1}\gamma\right)}{\sin\left(\lambda_j^{(\sigma)} - \lambda_{j'}^{(\sigma+1)} + \frac{\mathrm{i}}{2}\epsilon_{\sigma+1}\gamma\right)} \tag{15}$$

where

$$\begin{aligned} &j = 1, 2, \ldots, m_\sigma \qquad \sigma = 0, 1, \ldots, q-1 \\ &\lambda_j^{(-1)} = 0 \qquad \lambda_j^{(0)} = \lambda_j \\ &n_j = m_{j-1} - m_j \qquad m_{-1} = L \qquad m_q = 0 \qquad n_0 = n \end{aligned} \tag{16}$$

and

$$\begin{aligned} &q = P \qquad \epsilon_0 = 1 \qquad G_{\sigma\sigma} = G_{0\sigma} = G_{\sigma 0} = 1 \\ &\epsilon_i = \varepsilon_i \qquad G_{ik} = g_{ik} \qquad (i, k = 1, 2, \ldots, P) \end{aligned} \tag{17}$$

n_j being the number of particles in the class Q_j.

The total energy E and momentum P of the model are given in terms of the particle rapidities λ_j,

$$\begin{aligned} E &= -2\sum_{j=1}^{n} \cos k_j = 2\varepsilon_0 \sum_{j=1}^{n} \left(\cosh\gamma \frac{\sinh^2\gamma}{\cosh\gamma - 2\cos 2\lambda_j}\right) \\ P &= \sum_{j=1}^{n} k(\lambda_j). \end{aligned} \tag{18}$$

Consider now the model (4) in the general case where in each class Q_j we have n_j unpaired particles and n'_j conjugate pairs of particles. The reference state Ψ_0 is made up of a state in which there are no conjugate pairs. Examining (4) we see that when this Hamiltonian acts on a state, it looks for conjugate pairs and replaces them by a sum over all such pairs from a given set Q_j. The second possible process is the permutation of two neighbour particles or one particle and one conjugate pair. It is important that both

processes do not depend on the number of types of particles in each set N_{Q_j}. Thus, the Bethe ansatz equations depend only on n_j and $n'_j (j = 1, 2, \ldots, Q_j)$.

We now restrict ourselves to the case where we have arbitrary n_j and $n'_j (j = 1, 2, \ldots, Q)$ but only all $N_{Q_j} = 2$. We have shown by a direct calculation that in this case Hamiltonian (4) can be reduced to the $2P$-component t–J model [18, 20]. Thus, the general solution of the model (4) in this case is given by (15)–(17) with redefined parameters ε_σ and $G_{\sigma\rho}$

$$\begin{aligned} &q = 2P \qquad \epsilon_0 = 1 \qquad G_{\sigma\sigma} = G_{0\sigma} = G_{\sigma 0} = 1 \qquad \epsilon_i = \varepsilon_{[(i+1)/2]} \\ &G_{ik} = g_{[(i+1)/2],[(k+1)/2]} \qquad [(i+1)/2] \neq [(k+1)/2] \qquad (i, k = 1, 2, \ldots, 2P) \\ &G_{2l-1,2l} = G_{2l,2l-1} = \varepsilon_l \qquad (l = 1, 2, \ldots, P) \end{aligned} \tag{19}$$

and now n_{2l-1} and $n_{2l} - n_{2l-1}$ are numbers of conjugate pairs and separate particles of the set $Q_l (l = 1, 2, \ldots, P)$.

The ground-state energy and excited states of the Hamiltonian (4) can be calculated in principle by straightforward methods on the base of Bethe ansatz equations (15)–(17), which in the thermodynamic limit can be written as integral equations. However, the solution of these equations depends strongly on sign functions ϵ_α as well as on the phase factors g_{ik} and should be considered separately for different choices of these parameters.

Since these models satisfy the Hecke algebras we expect for free boundary conditions that their eigenspectra should depend only on the values of the variables ε_i and g_{ik}; $(i, k = 1, 2, \ldots, P)$, apart from degeneracies [31].

In the case of periodic boundary conditions this algebraic argument does not work and in general the eigenspectra will also depend on the particular choice of the classes Q_j. However, in the thermodynamic limit these differences should be irrelevant and we expect that the solution (15)–(18) will be valid for the arbitrary N_{Q_j} for a given set of $\{\epsilon_i\}$ and $\{g_{ik}\}$.

This work was supported in part by the Brazilian agencies CNPq, FAPESP, FINEP and by The Russian Foundation of Fundamental Investigations (Grant No RFFI 97-02-16146).

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